Asymptotics of soliton solution for the perturbed Davey-Stewartson-1 equations*

O.M.Kiselev,
Institute of Mathematics,
Ufa Sci Centre of Russian Acad. of Sci
112, Chernyshevsky str., Ufa, 450000, Russia
E-mail: ok@ imat.rb.ru

October 14, 1999

Abstract

The dromion of the Davey-Stewartson-1 equation is studied under perturbation on the large time.

In this work we construct the asymptotic solution of the Davey-Stewartson-1 equations (DS-1):

$$i\partial_t Q + \frac{1}{2}(\partial_\xi^2 + \partial_\eta^2)Q + (G_1 + G_2)Q = \varepsilon i F,$$

$$\partial_\xi G_1 = -\frac{\sigma}{2}\partial_\eta |Q|^2, \quad \partial_\eta G_2 = -\frac{\sigma}{2}\partial_\xi |Q|^2. \tag{1}$$

Here ε is small positive parameter, F is operator of the perturbation, the values of the parameter $\sigma = \pm 1$ correspond so called focusing or defocusing DS-1 equations.

The equations (1) at $\varepsilon = 0$ describe the interaction of long and short waves on the liquid surface if the capillary effects and potential flow are taken into account [1, 2]. The theorems about the existence of the solutions for this equations in the different functional classes are known [3, 4]. The inverse scattering transform method for the DS equations was formulated in [5]–[8]. This method allows to construct the soliton solutions [9] and to study the global properties of the solutions, for instance, the asymptotic

---

*This work was supported by RFBR 97-01-00459

1

behavior at large time [10, 11]. The asymptotic behavior of the solutions for nonintegrable DS equations was studied in [12].

The perturbed equations (1) are nonintegrable by inverse scattering transform method. Explicit form of the perturbation operator can be defined by small irregularity of bottom or by taking into account the next corrections in more realistic models for the liquid surface considered in [1, 2]. As main example in this work we use the perturbation appearing because of small irregularity of the bottom: $F \equiv AQ$. Here A is real constant. The sign $(\pm)$ of A corresponds to decreasing or increasing depth with respect to spatial variable ξ. Here we don't discuss the other perturbations corresponding to the system of liquid surface though the method we used allows to study the DS-1 equations with more wide class of the perturbation operators.

The special solution of DS-1 we perturbed here was constructed in the work [9]:

$$
q(\xi, \eta, t) = \frac{\rho \lambda \mu \exp(it(\lambda^2 + \mu^2))}{2 \cosh(\mu \xi) \cosh(\lambda \eta)(1 - \sigma |\rho|^2 \frac{\mu \lambda}{16}(1 + \tanh(\lambda \eta))(1 + \tanh(\mu \xi)))},
\tag{2}
$$

where λ, μ are positive constants defined by boundary conditions as $\eta \to -\infty$ and $\xi \to -\infty$; ρ is free complex parameter of the solution.

This solution decreases with respect to spatial variables exponentially. Besides if $\sigma = 1$ and $\frac{\mu \lambda}{4}|\rho|^2 > 1$ this solution has singularities on some lines $\eta = \text{const}$ and $\xi = \text{const}$.

The solution (2) was called dromion in work [7]. The inverse scattering method for dromion-like solution of (1) was developed in [7].

The constructing of the the asymptotic soliton-like solutions for the perturbations for the integrable equations is popular field. The perturbations of the (1+1)-dimensional integrable equations are investigated in more detail. In that case the asymptotic solutions are usable at long time and the Fourier-like integral corrections were studied in [13]-[21]. The instability of the solutions of the (2+1)-dimensional equations with respect to transversal perturbations was studied for example in [22]-[24]. But the perturbation of the essential two-dimensional solutions with respect to spatial variables of the (2+1)-dimensional integrable equations is less developed field. First cause is more diversity of the solutions and second cause is more bulky formulas for the solutions. Here we must refer the works [25]-[27]. In these works the perturbation of the soliton solution for the DS-2 equations had been studied. From these results and from the work about the asymptotic behavior of non-soliton solutions for the DS-2 equations [10] it follows that the soliton of the DS-2 equations is instable with respect to small perturbation of the initial data. These results stimulate the studies of the dromion perturbation.

The possibility of the full investigation of the linearized DS-1 equations plays the main role to construct the perturbation theory of the nonlinear DS-1 equations. The linearizations of the integrable equations have basic set of solutions usually. It allows to solve the linearized equations by Fourier method. For (1+1)-dimensional equations it was shown by Kaup in [28]. The works [29]-[31] were devoted to the same questions. The basic sets of the solutions for the linearized DS-2 and DS-1 equations were obtained in [32], [33].

1 Problem and result

We construct the asymptotic solution of equations (1) on $\mod(O(\varepsilon^2))$ uniformly at large t. The perturbation operator is $F \equiv AQ$ and the boundary conditions for G_1 and G_2:

$$G_1|_{\xi \to -\infty} = u_1 \equiv \frac{\lambda^2}{2\cosh^2(\lambda\eta)}, \quad G_2|_{\eta \to -\infty} = u_2 \equiv \frac{\mu^2}{2\cosh^2(\mu\xi)}. \qquad (3)$$

We find the asymptotic solution in the form:

$$Q(\xi,\eta,t,\varepsilon) = W(\xi,\eta,t,\tau) + \varepsilon U(\xi,\eta,t,\tau),$$
$$G_1(\xi,\eta,t,\varepsilon) = g_1(\xi,\eta,t,\tau) + \varepsilon h_1(\xi,\eta,t,\tau),$$
$$G_2(\xi,\eta,t,\varepsilon) = g_2(\xi,\eta,t,\tau) + \varepsilon h_2(\xi,\eta,t,\tau), \qquad (4)$$

where $\tau = \varepsilon t$ is slow time. The leading term of the asymptotics has the form:

$$W(\xi,\eta,t,\tau) = q(\xi,\eta,t;\rho(\tau)),$$

and g_1, g_2 are:

$$g_1(\xi,\eta,t,\tau) = u_1 - \frac{\sigma}{2} \int_{-\infty}^{\xi} d\xi' \partial_\eta |W(\xi',\eta,t,\tau)|^2,$$
$$g_2(\xi,\eta,t,\tau) = u_2 - \frac{\sigma}{2} \int_{-\infty}^{\eta} d\eta' \partial_\xi |W(\xi,\eta',t,\tau)|^2.$$

Denote $\gamma(\tau) = 1 - \sigma \frac{\mu\lambda}{4} |\rho(\tau)|^2$, $\gamma_0 = \gamma(0)$.

Theorem 1. *If*

$$\gamma(\tau) = \gamma_0^{\exp(2A\tau)}, \quad Arg(\rho(\tau)) \equiv const,$$

where $\gamma_0 > 1$ at $\sigma = -1$ and $0 < \gamma_0 < 1$ at $\sigma = 1$, then the asymptotic solution (4) with respect to $\mod(O(\varepsilon^2))$ is useful uniformly at $t = O(\varepsilon^{-1})$.

3

Remark 1. At more long time $t \ll \varepsilon^{-1} \log(\log(\varepsilon^{-1}))$ the formulas (4) are asymptotic solution of (1) with respect to $\mathrm{mod}(o(1))$ only.

The asymptotic analysis given here is valid for the solutions (2) without the singularities. It means if $\sigma = 1$, then $\frac{\mu\lambda}{4}|\rho(\tau)|^2 < 1$ or the same $\gamma(\tau) > 0$. If the coefficient of the perturbation $A > 0$, then $|\rho|$ increases ($\gamma \to 0$ at $\tau \to \infty$) with respect to slow time. It allows to say that the singularity may appear in the leading term of the asymptotics at $\tau \to \infty$. However we can't state this strongly because our asymptotics is usable only at $\tau \ll \log(\log(\varepsilon^{-1}))$, while the order of $\gamma(\tau)$ is greater than ε. The order of a remainder term after substituting the asymptotic solution (4) into the equations (1) is only $o(1)$ at the very long time $1 \ll \tau \ll \log(\log(\varepsilon^{-1}))$.

The obtained result for the problem about the interaction of the long and short waves on the liquid surface shows that when the depth decreases the the formal asymptotic solution can be described by adiabatic perturbation theory of the dromion for the focusing DS-1 equations at least for $|t| \ll \varepsilon^{-1} \log(\log(\varepsilon^{-1}))$.

Remark 2. The appearance of the singularities in the solution of non-integrable cases of the Davey-Stewartson equations is known phenomenon [34].

Remark 3. D. Pelinovsky notes to author, that the modulation of the parameter $|\rho|$ may be obtained out of the "energetic equality" [35]:

$$\partial_t \int \int_{\mathbb{R}^2} d\xi d\eta |Q|^2 = \varepsilon \int \int_{\mathbb{R}^2} d\xi d\eta (Q\bar{F} - \bar{Q}F).$$

2 Solution of linearized equations

Here we remind the formulas for solution of the linearized DS-1 equations on the dromion as a background:

$$i\partial_t U + (\partial_\xi^2 + \partial_\eta^2)U + (G_1 + G_2)U + (G_1 + G_2)Q = iF \tag{5}$$

$$\partial_\xi V_1 = -\frac{\sigma}{2}\partial_\eta(Q\bar{U} + \bar{Q}U), \quad \partial_\eta V_2 = \frac{-\sigma}{2}\partial_\xi(Q\bar{U} + \bar{Q}U).$$

The results of inverse scattering transform [7] and the set of the basic functions [33] are used for solving of the linearized equations by Fourier method. However unlike the work [33] here the solution of the DS-1 equations with nonzero boundary conditions is considered. It leads to the change of the dependency of the scattering data with respect to time (see also [7]) and of the formulas which define the dependency of Fourier coefficients of the solution for the linearized DS-1 equations in contrast to obtained in [33].

In the inverse scattering transform one use the matrix solution of the Dirac system to solve the DS-1 equation [5]-[7]:

$$\begin{pmatrix} \partial_\xi & 0 \\ 0 & \partial_\eta \end{pmatrix} \psi = -\frac{1}{2} \begin{pmatrix} 0 & Q \\ \sigma \bar{Q} & 0 \end{pmatrix} \psi. \tag{6}$$

Let ψ^+ and ψ^- are the matrix solutions of the Goursat problem for the Dirac system with the boundary conditions ([7])

$$\begin{array}{cc} \psi_{11}^+|_{\xi \to -\infty} = \exp(ik\eta), & \psi_{12}^+|_{\xi \to -\infty} = 0, \\ \psi_{21}^+|_{\eta \to \infty} = 0, & \psi_{22}^+|_{\eta \to -\infty} = \exp(-ik\xi); \\ \psi_{11}^-|_{\xi \to -\infty} = \exp(ik\eta), & \psi_{12}^-|_{\xi \to \infty} = 0, \\ \psi_{21}^-|_{\eta \to -\infty} = 0, & \psi_{22}^-|_{\eta \to -\infty} = \exp(-ik\xi). \end{array} \tag{7}$$

Denote by $\psi_{(j)}^+$, $j = 1, 2$, the column of the matrix ψ^+, then this column is the solution of two systems. There are the system (6) and additional system of time evolution:

$$\partial_t \psi_{(1)}^+ = ik^2 \psi_{(1)}^+ + i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} (\partial_\xi - \partial_\eta)^2 \psi_{(1)}^+$$

$$+i \begin{pmatrix} 0 & Q \\ \sigma \bar{Q} & 0 \end{pmatrix} (\partial_\xi - \partial_\eta) \psi_{(1)}^+ + \begin{pmatrix} iG_1 & -i\partial_\eta Q \\ i\sigma \partial_\xi \bar{Q} & -iG_2 \end{pmatrix} \psi_{(1)}^+. \tag{8}$$

One can obtain the equation like this for the other columns of the matrices $\psi^\pm$. These equations are differ from (8) by the sign of ik^2 in first term of the right hand side only.

Now we write two bilinear forms defining analogs of the direct and inverse Fourier transforms. First bilinear form is

$$(\chi, \mu)_f = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\xi d\eta (\chi_1 \mu_1 \sigma \bar{f} + \chi_2 \mu_2 f). \tag{9}$$

Here χ_i and μ_i are the elements of the columns χ and μ.

Denote by $\phi_{(1)}$ and $\phi_{(2)}$ the solutions conjugated to $\psi_{(1)}^+$ and $\psi_{(2)}^-$ with respect to the bilinear form (9).

Using the formulas for the scattering data ([7]) one can write these data:

$$s_1(k, l) = \frac{1}{4\pi} (\psi_{(1)}^+(\xi, \eta, k), E_{(1)}(il\xi))_Q, \tag{10}$$

$$s_2(k, l) = \frac{1}{4\pi} (\psi_{(2)}^-(\xi, \eta, k), E_{(2)}(il\eta))_Q. \tag{11}$$

Here $E(z) = \mathrm{diag}(\exp(z), \exp(-z))$.

5

It is shown ([7]) the elements of the matrices $\psi^\pm$ are analytic functions with respect to the variable k as $\pm Im(k) > 0$. Using the scattering data one can write the nonlocal Riemann-Hilbert problem for the ψ_{11}^- and ψ_{12}^+ on the real axes ([7]):

$$\psi_{11}^-(\xi,\eta,k) = \exp(ik\eta) + \exp(ik\eta)\left(\exp(-ik\eta) \int_{-\infty}^\infty dl\, s_1(k,l)\psi_{12}^+(\xi,\eta,l)\right)^-,$$

$$\psi_{12}^+(\xi,\eta,k) = \exp(-ik\xi)\left(\exp(ik\xi) \int_{-\infty}^\infty dl\, s_2(k,l)\psi_{11}^-(\xi,\eta,l)\right)^+.$$

Here

$$\left(f(k)\right)^\pm = \frac{1}{2i\pi} \int_{-\infty}^\infty \frac{dk'\, f(k')}{k' - (k \pm i0)}.$$

The Riemann-Hilbert problem for ψ_{21}^- and ψ_{22}^+ has the form:

$$\psi_{21}^-(\xi,\eta,k) = \exp(ik\eta)\left(\exp(-ik\eta) \int_{-\infty}^\infty dl\, s_1(k,l)\psi_{22}^+(\xi,\eta,l)\right)^-,$$

$$\psi_{22}^+(\xi,\eta,k) = \exp(-ik\xi) +$$
$$\exp(-ik\xi)\left(\exp(ik\xi) \int_{-\infty}^\infty dl\, s_2(k,l)\psi_{21}^-(\xi,\eta,l)\right)^+.$$

Introduce second bilinear form

$$\langle \chi, \mu \rangle_s = \int_{-\infty}^\infty \int_{-\infty}^\infty dk dl (\chi^1(l)\mu^1(k)\, s_2(k,l) + \chi^2(l)\mu^2(k)\, s_1(k,l)), \qquad (12)$$

where χ^j is the element of the row χ.

Denote by $\varphi^{(j)}$, $j = 1,2$, the row conjugated to $\psi^{(j)} = [\psi_{j1}^-, \psi_{j2}^+]$ with respect to the bilinear form (12). Formulate the result about the decomposition obtained in [33].

Theorem 2. *Let Q be such that $\partial^\alpha Q \in L_1 \cap C^1$ for $|\alpha| \le 3$, if $f(\xi,\eta)$ is $\partial^\alpha f \in L_1 \cap C^1$ for $|\alpha| \le 4$, then one can write f in the form*

$$f = \frac{-1}{\pi}\langle \psi^{(1)}(\xi,\eta,l), \varphi^{(1)}(\xi,\eta,k)\rangle_{\hat{f}},$$

where

$$\hat{f} = \frac{1}{4\pi}(\psi_{(1)}^+(\xi,\eta,k), \phi_{(1)}(\xi,\eta,l))_f.$$

Using the theorem 2 one can solve the Cauchy problem for the linearized DS-1 equations. Here the dependence of $\hat{f}$ with respect to t is differ from the same obtained in [33] because here we use the DS-1 equations with nontrivial boundary conditions (3). These changes may be obtained using the results of [7] and [33].

6

Theorem 3. *Let Q be the solution of the DS-1 equations with the boundary conditions $G_1|_{\xi \to -\infty} = u_1$ and $G_2|_{\eta \to -\infty} = u_2$, and Q satisfy the conditions of theorem 2, the solution of the first of the linearized DS-1 equation is smooth and integrable function U with respect to ξ and η, where $\partial^\alpha U \in L_1 \cap C^1$ and $\partial^\alpha F \in L_1 \cap C^1$, for $|\alpha| \leq 4$ and $t \in [0, T_0]$. Then*

$$\partial_t \hat{U} = i(k^2 + l^2)\hat{U} + \int_{-\infty}^{\infty} dk' \hat{U}(k - k', l, t)\chi(k') +$$

$$\int_{-\infty}^{\infty} dl' \hat{U}(k, l - l', t)\kappa(l') + \hat{F}, \tag{13}$$

If the boundary conditions in the problem for the DS-1 equation are zero, then $\chi \equiv \kappa \equiv 0$. In this case the formulas of the theorem 3 allow to solve the linearized DS-1 equation in the explicit form. In this work we consider the solution of the DS-1 equation with nonzero boundary conditions. It leads to integral terms in the formulas of theorem 3. In order to solve the linearized DS-1 equation we must transform the formula (13). In the right hand side of (13) the integral terms are the convolutions. Go over to the equations for the Fourier transform of the functions $\hat{U}(k, l, t, \tau)$ with respect to variables k and l. As result we obtain the linear Schrödinger equation:

$$i\partial_t \tilde{U} + (\partial_\xi^2 + \partial_\eta^2)\tilde{U} + (u_2(\xi\mu) + u_1(\eta\lambda))\tilde{U} = \tilde{F}. \tag{14}$$

Here

$$\tilde{U}(\xi, \eta, t, \tau) = \frac{1}{2\pi} \int_{\mathbb{R}^2} dk dl \hat{U}(k, l, t, \tau) \exp(-ik\eta - il\xi);$$

$$\tilde{F}(\xi, \eta, t, \tau) = \frac{1}{2\pi} \int_{\mathbb{R}^2} dk dl \hat{F}(k, l, t, \tau) \exp(-ik\eta - il\xi).$$

The same equation without the right hand side was obtained in [7] for the time evolution of the scattering data for the nonlinear DS-1 equation.

One can obtain the solution of the Cauchy problem with zero in the initial conditions for the equations (14) by the separation of the variables. In our case the solution of the equations (14) obtained by the Fourier method has the form:

$$\tilde{U}(\xi, \eta, t) = \frac{1}{2\pi} \int_{\mathbb{R}^2} dm dn \breve{U}(m, n, t) X(\xi, m) Y(\eta, n) \exp(-it(m^2 + n^2)) +$$

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} dn \breve{U}_\mu(n, t) Y(\eta, n) X_\mu(\xi) \exp(-it(n^2 - \mu^2)) +$$

$$\frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} dn \breve{U}_\lambda(m, t) X(\xi, m) Y_\lambda(\eta) \exp(-it(m^2 - \lambda^2)) +$$

$$\breve{U}_{\mu,\lambda} X_\mu(\xi) Y_\lambda(\eta) \exp(it(\mu^2 + \lambda^2)).$$

7

Here we use notations:

$$X(m, \xi) = \frac{\mu \tanh(\mu \xi) + im}{im - \mu} \exp(-im\xi), \quad X_\mu(\xi) = \frac{1}{2\cosh(\mu\xi)};$$

$$Y(n, \eta) = \frac{\lambda \tanh(\lambda \eta) + in}{in - \lambda} \exp(-in\eta), \quad Y_\lambda(\eta) = \frac{1}{2\cosh(\lambda\eta)};$$

$$\partial_t \breve{U} = i(m^2 + n^2)\breve{U} + \breve{F}(m, n, t), \quad \partial_t \breve{U}_\mu = i(n^2 - \mu^2)\breve{U} + \breve{F}_\mu(n, t),$$
$$\partial_t \breve{U}_\lambda = i(m^2 - \lambda^2)\breve{U} + \breve{F}_\lambda(m, t), \quad \partial_t \breve{U}_{\mu\lambda} = -i(\lambda^2 - \mu^2)\breve{U} + \breve{F}_{\mu\lambda}(t),$$
$$\breve{U}|_{t=0} = \breve{U}_\mu|_{t=0} = \breve{U}_\lambda|_{t=0} = \breve{U}_{\mu\lambda}|_{t=0} = 0;$$

$$\breve{F}(m, n, t) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t) \bar{X}(m, \xi) \bar{Y}(n, \eta),$$

$$\breve{F}_\mu(n, t) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t) \bar{X}_\mu(\xi) \bar{Y}(n, \eta),$$

$$\breve{F}_\lambda(m, t, \tau) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t) \bar{X}(m, \xi) \bar{Y}_\lambda(\eta),$$

$$\breve{F}_{\mu\lambda}(t, \tau) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t) \bar{X}_\mu(\xi) \bar{Y}_\lambda(\eta).$$

3 The equation for first correction

In this part the equation for the slow modulation of the parameter $\rho(\tau)$ is obtained. This equation is necessary and sufficient condition for the uniform boundedness of the first correction of the expansion (4).

Substitute the formula (4) into the equations (1). Equate the coefficients with the same power of ε. The equations at ε^0 are realized because W, g_1, g_2 are the asymptotic solution of nonperturbed DS-1 equations. For the first correction we obtain the linearized DS-1 equations:

$$i\partial_t U + (\partial_\xi^2 + \partial_\eta^2)U + (g_1 + g_2)U + (V_1 + V_2)W = iH$$

$$\partial_\xi V_1 = -\frac{\sigma}{2}\partial_\eta(W\bar{U} + \bar{W}U), \quad \partial_\eta V_2 = \frac{-\sigma}{2}\partial_\xi(W\bar{U} + \bar{W}U), \qquad (15)$$

where

$$H = AW - \partial_\tau W.$$

Before to use the formulas from the previously section we reduce the form of the right hand side in first of the equations (15). In the leading term the parameter ρ depends on the τ only. The other parameters depend only on the boundary conditions and do not change under perturbation. For convenience

we represent $\rho(\tau) = r(\tau)\exp(i\alpha(\tau))$. The derivation of W with respect to slow variable τ can be written as:

$$\partial_\tau W = \partial_r W r' + \partial_\alpha W \alpha'.$$

Here the derivatives r' and α' are unknown. They will be obtain below.

Compute the function $\hat{H}$. From the theorem 2 and formulas of the functions ψ_+ and ϕ (see Appendix) we obtain:

$$\hat{H}(k,l,t,\tau) = \exp(-it(\lambda^2 + \mu^2))\Big(P(k,l;\rho) - R(k,l;\rho)\partial_\tau\rho\Big),$$

where

$$P(k,l;\rho) = \exp(it(\lambda^2 + \mu^2))\widehat{AW}, \quad R(k,l;\rho) = \exp(it(\lambda^2 + \mu^2))\widehat{\partial_\tau W}.$$

In these formulas we write the dependence on time in explicit form. It allows to remove the secular terms in the asymptotic solution (4). The differential equations for $\breve{U}$ have the forms:

$$\partial_t \breve{U} = i(m^2 + n^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}(m,n;\rho) - \breve{R}(m,n;\rho)),$$
$$\partial_t \breve{U}_\mu = i(n^2 - \mu^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}_\mu(n;\rho) - \breve{R}_\mu(n;\rho)),$$
$$\partial_t \breve{U}_\lambda = i(m^2 - \lambda^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}_\lambda(m;\rho) - \breve{R}_\lambda(m;\rho)),$$
$$\partial_t \breve{U}_{\mu\lambda} = -i(\lambda^2 - \mu^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}_{\mu\lambda}(\rho) - \breve{R}_{\mu\lambda}(\rho)).$$

One can see that the secular terms may appear because of the last term in the equation for $\breve{U}_{\mu\lambda}$. The requirement of equivalent to zero of this term leads us to the equation for $\rho(\tau)$:

$$\breve{R}_{\mu\lambda} - \breve{P}_{\mu\lambda} = 0, \quad \rho|_{\tau=0} = \rho_0. \tag{16}$$

In the result $\breve{U}_{\mu\lambda} \equiv 0$. The other equations for $\breve{U}$ are easy to integrate. The solutions of this equations are bounded on the all of arguments for all time.

We must return to the original of the images $\breve{U}$ to state about boundedness of the solution U, V_1, V_2 for the equations (15). One can see the direct (from U into $\breve{U}$) and inverse (from $\breve{U}$ into U) integral transforms as the Fourier transform from the smooth and exponentially decreasing functions with respect to the corresponding variables. The Fourier transform moves such functions into analytic functions near the real axis. The inverse transform moves these analytic functions into the exponential decreasing functions. So the solution of (15) is boundedness and decreasing exponentially with respect to the spatial variables.

9

4 Modulation equation for $\rho(\tau)$

Here the equation (16) for the parameter $\rho(\tau)$ is reduced to the more convient form. Write the derivative of the leading term with respect to the slow variable τ.

$$i\partial_\tau W = -\alpha' W - iW\frac{r'}{r} + \frac{2iW}{1 - \sigma r^2\frac{\mu\lambda}{16}(1 + \tanh(\mu\xi))(1 + \tanh(\lambda\eta))}\frac{r'}{r}.$$

Denote $\Gamma = \frac{\mu\lambda}{4}r^2$ and compute the images $(\check{\cdot})_{\mu\lambda}$ of every term.

$$(\check{W})_{\mu\lambda} = \frac{\sigma\bar{\rho}\exp(-it(\lambda^2 + \mu^2))}{8}(\sigma\Gamma - 1)\log|1 - \sigma\Gamma|;$$

$$(i\check{W})_{\mu\lambda} = -i\sigma\Gamma\frac{\rho\exp(-it(\lambda^2 + \mu^2))}{8}.$$

Denote the image of the last term by $\check{h}_{\mu\lambda}$. Its image has the form:

$$\check{h}_{\mu\lambda} = \frac{r'}{r}\frac{\sigma\bar{\rho}\exp(-it(\lambda^2 + \mu^2))}{8}(1 - \sigma\Gamma)\left(\frac{1}{1 - \sigma\Gamma} - 1 - \log|1 - \sigma\Gamma|\right).$$

The image $(\check{\cdot})$ of AW has the similar form.

Substitute these formulas into (16) and separate real and imaginary parts of this equation, then

$$\alpha' = 0,$$

$$\frac{r'}{r}(\sigma\Gamma - 1)\log|1 - \sigma\Gamma| - \frac{r'}{r}\left(\sigma\Gamma - (1 - \sigma\Gamma)\log|1 - \sigma\Gamma|\right) +$$
$$A(\sigma\Gamma - 1)\log|1 - \sigma\Gamma| = 0.$$

Use the notation for Γ, then the second equation has the form:

$$\frac{d\Gamma}{d\tau} = -2\sigma A(1 - \sigma\Gamma)\log|1 - \sigma\Gamma|.$$

This equation defines the evolution of the absolute value of the complex parameter ρ. The argument of this parameter do not change under the perturbation $F = AQ$.

Solve the equation for Γ. Denote $\gamma = 1 - \sigma\Gamma$, rewrite the equation for γ, then we obtain:

$$\gamma' = 2A\gamma\log(\gamma).$$

The solution for this equation has the form:

$$\gamma(\tau) = \exp(C\exp(2A\tau)),$$

10

where $\gamma|_{\tau=0} = \exp(C)$, then we can write the $\gamma(\tau)$ in the form:

$$\gamma(\tau) = \gamma_0^{\exp(2A\tau)}.$$

The theorem 3 is proved.

Acknowledgements

I thank D.Pelinovsky for the discussions of the results and for the helpful remarks.

5 Appendix

5.1 Explicit formulas

Here we remain the explicit forms of the solution for the Dirac equation with the dromion-like potential. These forms were obtained in [7]. In our computations we use first column of the matrix ψ^+ only.

$$\begin{pmatrix} \psi_{11}^+ \\ \psi_{21}^+ \end{pmatrix} = \begin{pmatrix} \exp(ik\eta) \\ 0 \end{pmatrix} + \frac{\int_\eta^\infty \frac{dp\lambda\exp(ikp))}{2\cosh(\lambda p)}}{(1 - \sigma|\rho|^2\frac{\lambda\mu}{16}(1 + \tanh(\mu\xi))(1 + \tanh(\lambda\eta)))}$$

$$\times \begin{pmatrix} \frac{-\sigma|\rho|^2\lambda\mu(1+\tanh(\mu\xi))}{8\cosh(\lambda\eta)} \\ \frac{\sigma\bar\rho\mu\exp(-it(\lambda^2+\mu^2))}{2\cosh(\mu\xi)} \end{pmatrix}. \tag{17}$$

5.2 Solutions of conjugated equations

Here we write the problems for the functions conjugated to $\psi^\pm$ with respect to the bilinear forms.

The functions ϕ are the solutions of the boundary problem conjugated to the solutions of the problem (6), (3) with respect to the bilinear form (9). First column of the matrix ϕ is the solution of the integral equation:

$$\phi_{11}(\xi,\eta,l,t) = \exp(il\xi) + \frac{1}{2}\int_{-\infty}^\eta d\eta' Q(\xi,\eta',t)\phi_{21}(\xi,\eta',l,t),$$

$$\phi_{21}(\xi,\eta,l,t) = \frac{-1}{2}\int_\xi^\infty d\xi' \bar Q(\xi',\eta,t)\phi_{11}(\xi',\eta,l,t).$$

The explicit formulas for first column of the matrix ϕ used in section 4 for the dromion potential has the form:

$$\begin{pmatrix} \phi_{11} \\ \phi_{21} \end{pmatrix} = \begin{pmatrix} \exp(il\xi) \\ 0 \end{pmatrix} + \frac{\int_\xi^\infty \frac{dp\mu\exp(ikp))}{2\cosh(\mu p)}}{(1 - \sigma|\rho|^2\frac{\lambda\mu}{16}(1 + \tanh(\mu\xi))(1 + \tanh(\lambda\eta)))} \times$$

11

$$\times \begin{pmatrix} \frac{-\sigma|\rho|^2\lambda\mu(1+\tanh(\lambda\eta))}{8\cosh(\mu\xi)} \\ \frac{-\sigma\bar{\rho}\lambda\exp(-it(\lambda^2+\mu^2))}{2\cosh(\lambda\eta)} \end{pmatrix}. \tag{18}$$

Second bilinear form (12) allows to write the integral equations conjugated to integral equations which were obtained from the nonlocal Riemann-Hilbert equation for the functions $\psi^\pm$ in [7]. Using this equations for the functions $\varphi(\xi,\eta,l)$ one can show that the functions φ are the solutions of the boundary equations for the Dirac system or the integral equations:

$$\varphi_{11}(\xi,\eta,l) = \exp(il\xi) + \frac{1}{2}\int_{-\infty}^{\eta} d\eta' Q(\xi,\eta',t)\varphi_{21}(\xi,\eta',l),$$

$$\varphi_{21}(\xi,\eta,l) = \frac{1}{2}\int_{-\infty}^{\xi} d\xi' \bar{Q}(\xi',\eta,t)\varphi_{11}(\xi',\eta,l).$$

The functions φ have the similar form as the functions ϕ.

References

[1] A.Davey, K.Stewartson. On expansions on products of the solution for two Dirac systems. Proc.R.Soc.London. Ser.A. v.338, 1974, p.101–110.

[2] V.D.Djordjevic, L.G.Redekopp. On two-dimensional packets of capillary-gravity waves J.Fluid Mech. v.79, 1977, p.703–714.

[3] J.-M.Ghidaglia, J.-K.Saut. On the initial value problem for the Davey-Stewartson systems. Nonlinearity, v.3, 1990, p.475-506.

[4] M.M. Shakir'yanov. Solvability of the initial-boundary problem for the Davey-Stewartson type system of equations. In: Integrability in the dynamical systems. Inst. of Math. Ufa, 1994, p.49-61.

[5] L.P. Nizhnik. Inverse problems for hyperbolic equations. Naukova Dumka, Kiev, 1991, pp.232.

[6] A.S.Fokas, M.J.Ablowitz. On the inverse scattering transform of multidimensional nonlinear equations related to first-order systems in the plane. J. Math. Phys. 25(8), 1984, p.2494-2505.

[7] A.S.Fokas, P.M.Santini. Dromions and a boundary value problem for the Davey-Stewartson I equation. Physica D, 44(1990), p.99-130.

[8] A.S.Fokas, L.J.Sung. On the solvability on the N-wave, Davey-Stewartson and Kadomtsev-Petviashvili equations. Inv. probl., 8(1992), p.673-708.

[9] M. Boiti, J. JP. Leon, L. Martina, F. Pempinelli. Scattering of localized solitons in the plane. Phys. Lett. A32, 1988, p.432-439.

[10] O.M.Kiselev. Asymptotic behavior of solitonless solution of the Davey-Stewartson-2 equation. Differentsial'nye Uravneniya, 1997, v.33, n6, p.812-819.

[11] O.M.Kiselev. An asymptotic solution of the Cauchy problem for the Davey-Stewartson-1 equation. Teor. Matem. Fiz., 1998, v.114, n1, p.104-114.

[12] H.Nakao, H.Hirata. Global existence and asymptotic behavior in time of small solutions to the elliptic-hyperbolic Davey-Stewartson system. Nonlinearity,1996, v.9, c.1387-1409.

[13] D.J.Kaup. A perturbation expansion for the Zakharov-Shabat inverse scattering transform. SIAM J.on Appl.Math., 1976, v. 31, p. 121–133.

[14] V.I.Karpman, E.I. Maslov. The structure of tails, appearing under soliton perturbations. JETPh, 1977, v.73, p. 537–559.

[15] J.R.Keener, D.J.McLaughlin. A Green's function for a linear equation associated with solitons. J.Math.Phys., 1977, v.18, p.2008.

[16] A.C.Newell. The inverse scattering transform. Topics in Current Phys., v. 17, 1980, New York, Springer-Verlag, p. 177–242.

[17] E.M. Maslov. To soliton perturbation theory in the second approach. Teor.Matem.Fiz.,, 1980, v.l42, p.362–373.

[18] V.P. Maslov, G.A.Omel'yanov. Asymptotic soliton-like solutions of equations with small dispersion. Uspekhi matem nauk, 1981, v. 36, n3, p.63–126.

[19] Y.S.Kivshar, B.A.Malomed. Solitons in nearly integrable systems. Rev. Mod. Phys., 1989, v. 61, n4, p.763–915.

[20] L.A.Kalyakin. Perturbation of K-dV soliton. Teor.Matem.Fiz., 1992, v.l92, p.62–77.

[21] O.M.Kiselev. Asymptotics of kink of perturbed sine-Gordon equation. Teor.Matem.Fiz., 1992, v.l93, p.39–48.

[22] V.V. Kadomtsev, V.I.Petviashvili. About stability of the solitary waves in the medium with small dispersion. DAN SSSR, 1970, v.192, p.753.

[23] L.A.Bourdag, A.R.Its, S.V.Manakov, V.B.Matveev, V.E.Zakharov. Two-dimensional solitons of the Kadomtsev-Petviashvili equation and their interactions. Phys.Lett. A, 1979, n3, p.205.

[24] D.E Pelinovsky, Yu.A.Stepanyants. Self-focusing instability of plane solitons in the medium with positive dispersion. JETPh, 1993, v.104, n1, p.3387-3340.

[25] R.R. Gadyl'shyn, O.M.Kiselev. On nonsoliton structure of scattering data under perturbation of two-dimensional soliton for Davey-Stewartson equation II. Teor.Matem.Fiz., 1996, v.106, n2, p.167-173.

[26] R.R.Gadylshin, O.M.Kiselev. Structural instability of soliton for Davey-Stewartson II equation. Teor.Matem.Fiz., 1999, v.118, n3, p.354-361.

[27] R.R.Gadyl'shin, O.M.Kiselev. Asymptotics of perturbed soliton solution for the Davey-Stewartson II equation. PBB /solv-int # 9801014.

[28] D.J.Kaup. Closure of the squared Zakharov-Shabat eigenstates. J.of Math.Anal.and Appl., 1976, v. 54, p.849.

[29] J.R.Keener, D.J.McLaughlin. A Green's function for a linear equation associated with solitons. J.Math.Phys., 1977, v.18, p.2008.

[30] V.S.Gergjikov, E.H.Hristov. On expansions on products of the solution for two Dirac systems. Matem. Zametki., 1980, v.28, p.501.

[31] R.L.Sachs. Completeness of derivatives of squared Schrödinger eigenfunction and explicit solutions of the linearized KdV equation. SIAM J.Math.Anal., 1983, v.14, p.674.

[32] O.M.Kiselev. Basic functions associated with two dimensional Dirac system. Func.Anal. Appl.,1998, v.32, n1, p.56-59.

[33] O.M.Kiselev. Perturbation theory for the Dirac equation in two-dimensional space. J.Math.Phys., 1998, v.39, p.2333-2345.

[34] G.S.Papanicolaou, C.Sulem, P.L.Sulem, X.P.Wang. The focusing singularity of the Davey-Stewartson equations for gravity-capillary surface waves. Physica D, 1994, v.72, p.61-86.

[35] D. Pelinovsky, private communication.